Qualitative solution of QCD sum rules

Sergey Afonin and Domenec Espriu

Departament d'Estructura i Constituents de la Matèria and
CER for Astrophysics, Particle Physics and Cosmology, Universitat de Barcelona
647 Diagonal, 08028, Spain
E-mail: `afonin@ecm.ub.es`, `espriu@ecm.ub.es`

Abstract: We show how such important features of QCD as chiral symmetry breaking or the formation of a mass-gap can be directly traced from QCD sum rules for two point functions assuming, in the large number of colors limit, exact duality between the operator product expansion and the spectrum described by linearly (or nearly linear) rising Regge trajectories as predicted by string theory. We see how the presence of chiral symmetry breaking is intimately related to confinement in this scenario, as expected from general arguments, and how Regge trajectories change when chiral symmetry is broken. As a result the whole meson mass spectrum can be parametrized with a good accuracy by the constant f_π only, thus realizing the program proposed by Migdal some time ago.

Contents

1. Introduction

In the present work we consider QCD sum rules for the vector (V), axial-vector (A), scalar (S), and pseudoscalar (P) channels in the large-N_c limit, the planar limit of QCD [1] and match the corresponding operator product expansions (OPE) with the predictions of the QCD effective string theory. This approach has been used by several authors previously, (see e.g. [2–7]). In a sense, this work may be partly viewed as a simplified version of the analysis performed in [2] but with a rather different accent.

First of all, we show how the parameters of the effective string mass spectrum can be extracted from the sum rules for the two-point functions in a very simple way and without involved numerical fits (this is the main distinction with [2]). Second, we try to give a clear physical interpretation of all results appearing in the analysis. We see how in this framework chiral symmetry breaking and confinement (i.e. the creation of a mass gap) are intimately related. Finally we provide in this rather well defined theoretical setting an implementation of Migdal's program whereby all physical quantities in theories like QCD are expressed, modulo simple numerical factors, in terms of a basic scale. f_π is this scale in the present case.

The paper is organized as follows. In section 2 we introduce the relevant formulas. Sections 3 and 4 are devoted to the analysis of linear string-like mass spectrum for vector and scalar channels correspondingly. In sections 5 and 6 this analysis is extended to the non-linear spectrum. The results obtained are interpreted in section 7. Section 8 is devoted to a comparison with previous results. Final section 9 contains some concluding remarks.

2. General formalism

Let us briefly remind the reader of the general formalism for deriving QCD sum rules in the large-N_c limit. In this limit the two-point correlation functions of quark currents are saturated by an infinite set of narrow meson states with the quantum numbers of these currents, i.e. they can be represented in Euclidean space as follows

$$\Pi^J(Q^2) = \int d^4x \, e^{iQx} \langle \bar{q}\Gamma q(x) \bar{q}\Gamma q(0) \rangle_{\text{planar}} = \sum_n \frac{Z_J(n)}{Q^2 + m_J^2(n)} + D_0^J + D_1^J Q^2, \qquad (2.1)$$

$$J \equiv S, P, V, A; \qquad \Gamma = i, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5; \qquad D_0, D_1 = \text{const.} \qquad (2.2)$$

For simplicity we do not write the Pauli matrices but the $SU_f(2)$ symmetry is understood. The last two terms are required for renormalization of infinite sums. On the other hand, their high-energy asymptotics are given by the OPE [8–10] (we consider the chiral limit)

$$\Pi^{V,A}(Q^2) = \frac{N_c}{12\pi^2} \ln\frac{\Lambda^2}{Q^2} + \frac{\alpha_s}{12\pi} \cdot \frac{\langle G^2 \rangle}{Q^4} + \frac{4\xi^{V,A}}{9}\pi\alpha_s\frac{\langle \bar{q}q \rangle^2}{Q^6}, \qquad (2.3)$$

$$\Pi^{S,P}(Q^2) = -\frac{N_c}{8\pi^2}Q^2 \ln\frac{\Lambda^2}{Q^2} + \frac{\alpha_s}{8\pi} \cdot \frac{\langle G^2 \rangle}{Q^2} - \frac{2\xi^{S,P}}{3}\pi\alpha_s\frac{\langle \bar{q}q \rangle^2}{Q^4}, \qquad (2.4)$$

where

$$\xi^{V,P} = -7, \qquad \xi^{A,S} = 11, \qquad (2.5)$$

and we have defined

$$\Pi_{\mu\nu}^{V,A}(Q^2) \equiv \left(-\delta_{\mu\nu}Q^2 + Q_\mu Q_\nu\right)\Pi^{V,A}(Q^2). \qquad (2.6)$$

The symbols $\langle G^2 \rangle$ and $\langle \bar{q}q \rangle$ denote the gluon and quark condensate, respectively. The residues are parametrized as follows

$$Z_{V,A}(n) \equiv 2F_{V,A}^2(n), \qquad Z_{S,P}(n) \equiv 2G_{S,P}^2(n)m_{S,P}^2(n). \qquad (2.7)$$

with $F_{V,A}(n)$ being electromagnetic decay constants and $G_P(n)$ are related with the corresponding weak decay constants (see [2] for details).

The sum rules simply follow from comparison at each power of Q^{-2} of the OPE (2.3) and (2.4) with the sum in Eq. (2.1) after summing up over resonances (in a chiral invariant way) and subtracting infinite constants which are irrelevant for our purposes.

3. Sum rules for linear spectrum: Vector case

Phenomenology tells us that the linear mass spectrum predicted by effective string theories (such as the ones represented by the Veneziano amplitude, the supersymmetric string, or the Lovelace-Shapiro amplitude) is a very good approximation to the real world [11]. In the present section we shall adhere to this type of spectrum, namely assume that the spectrum is strictly linear and see what are the implications of this assumption. Thus consider the ansatz

$$m_J^2(n) = m^2 + an, \qquad F^2(n) = \text{const}, \qquad n = 0, 1, 2, \ldots \qquad (3.1)$$

Here the universal slope a is proportional to the string tension T: $a = 2\pi T$. After renormalization (subtraction of infinite constant) and taking into account the π-meson contribution (or more generally, the possibility of a zero-mass state in the pseudoscalar and axial-vector channels) we have

$$\Pi^{V,A}(Q^2) = \frac{2f_\pi^2}{Q^2}\delta_{AJ} + \sum_{n=0}^{\infty}\frac{2F_{V,A}^2}{Q^2+m^2+an} = \frac{2f_\pi^2}{Q^2}\delta_{AJ} - \frac{2F_{V,A}^2}{a}\,\psi\left(\frac{Q^2+m^2}{a}\right), \quad (3.2)$$

where we have used the Kronecker symbol δ_{ab} to indicate that the hypothetical Goldstone boson contributes only to the axial-vector channel due to PCAC.

The ψ-function has an asymptotic representation at $z \gg 1$

$$\psi(z) = \ln z - \frac{1}{2z} - \sum_{k=1}^{\infty}\frac{B_{2k}}{2kz^{2k}}, \quad (3.3)$$

there B_{2k} denote the Bernoulli numbers.

Let us introduce the notation

$$x \equiv \frac{m^2}{a}. \quad (3.4)$$

Performing the procedure outlined above one arrives at the following sum rules

$$\frac{N_c}{24\pi^2} = \frac{F_J^2}{a}, \quad (3.5)$$

$$0 = f_\pi^2\delta_{AJ} - F_J^2\left(x - 1/2\right), \quad (3.6)$$

$$\frac{\alpha_s}{12\pi}\langle G^2\rangle = F_J^2 a\left(x^2 - x + 1/6\right), \quad (3.7)$$

$$\frac{4\xi^J}{9}\pi\alpha_s\langle\bar{q}q\rangle^2 = -\frac{2}{3}F_J^2 a^2 x\left(x - 1/2\right)\left(x - 1\right). \quad (3.8)$$

From Eq. (3.5) it follows that the quantities F_J^2 are directly expressed through the slope a. Let us substitute them into Eq. (3.8). The result is

$$-\frac{\xi^J}{N_c}\pi\alpha_s 16\pi^2\frac{\langle\bar{q}q\rangle^2}{a^3} = x\left(x - 1/2\right)\left(x - 1\right). \quad (3.9)$$

Now we note that the quantity $\langle\bar{q}q\rangle^2/a^3 \approx (220/1100)^6 \sim 10^{-4}$ is very small in the real world. Thus the theory has a natural small parameter. More precisely, since the quantities $\xi^J/3$ and $\pi\alpha_s$ are of order one, the small parameter is

$$16\pi^2\frac{\langle\bar{q}q\rangle^2}{a^3} \approx 0.01. \quad (3.10)$$

The existence of this small parameter in the sum rules seems to be tightly related to that of the chiral perturbation theory, i.e. the ratio squared of m_π to the masses of light mesons in other channels. A kind of "perturbation theory" can be developed here. In the zero order the sum rules become extremely simple providing nevertheless a good approximation to the real world. In the rest of the paper we will consider only this case.

In this approximation Eq. (3.9) takes the form

$$x \left(x - 1/2\right) \left(x - 1\right) \simeq 0, \qquad (3.11)$$

and only two dimensionful parameters appear: The slope a which parametrizes the strength of the gluon interaction and, hence, must be directly related to the gluon condensate $\langle G^2 \rangle$ (indeed we see that from (3.7)), and the π-meson weak decay constant f_π parametrizing chiral symmetry breaking[1].

Let us investigate the solutions of the above set of equations. First of all, equations (3.5)-(3.7) and (3.11) have one trivial solution (even without any approximations) corresponding to the case where the theory is weakly coupled (non-confining), without bound states and condensates: $F_J^2 = a = 0$ but the ratio in Eq. (3.5) is non-zero and give the parton-model result. All non-perturbative effects are absent in this case.

Let us now analyze the possibility of a nontrivial[2] chirally symmetric solution with $f_\pi = 0$ (here of course $\langle \bar{q}q \rangle = 0$ and Eq. (3.11) is strictly true). From Eq. (3.6) we have only one possibility, namely $x = 1/2$, which is consistent with Eq. (3.11). Hence the chirally symmetric linear spectrum would be

$$m_J^2(n) = \frac{a}{2} + an. \qquad (3.12)$$

Substituting the solution in Eq. (3.7) and making use of Eq. (3.5) we obtain a relation between the slope and the gluon condensate

$$\frac{\alpha_s}{\pi} \langle G^2 \rangle = -\frac{N_c a^2}{24\pi^2}, \qquad (3.13)$$

numerically resulting in $\frac{\alpha_s}{\pi} \langle G^2 \rangle \approx -(370\,\text{MeV})^4$ that is in agreement with the standard value in QCD, $\frac{\alpha_s}{\pi} \langle G^2 \rangle = (360 \pm 20\,\text{MeV})^4$, in absolute value, but not in the sign, which is an obvious inconsistency.

In fact the gluon condensate is proportional to the QCD vacuum energy ε_{vac} (see discussions in section 7). As QCD is an asymptotic free theory the vacuum energy turns out to be negative for the stable vacuum [13]. We draw attention to the quite interesting fact that the solution $x = 1/2$ is nothing but the exact minimum of the function[3] $\varepsilon_{\text{vac}} \sim -\langle G^2 \rangle \sim x^2 - x + 1/6$. Hence, in the vector channels the chirally symmetric case for the linear spectrum corresponds to the theory near the maximum of this function (since the sign is opposite to that of vacuum energy).

Consequently, the chirally symmetric solution we have just been exploring cannot exist for the linear spectrum if confinement holds. Thus, the property of confinement in the large-N_c limit automatically results in Chiral Symmetry Breaking (CSB), i.e. $f_\pi \neq 0$. This agrees with the Coleman-Witten theorem [14]: if confinement persists in the planar limit of QCD, chiral symmetry is inevitably broken. As we will see later, the analysis of

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[1]Throughout the paper we use the value of f_π in the chiral limit [12], $f_\pi = 87$ MeV.

[2]I.e. with a mass gap.

[3]The vacuum energy (or gluon condensate) in our approach turns out to be the function of intercept, which is quantized, i.e. this energy is also quantized.

the chirally broken case necessarily requires the introduction of non-linear corrections to the spectrum in any case.

Staying within the framework of the linear spectrum, we note that the chirally symmetric spectrum (3.12) for the case of ρ-mesons agrees with the one derived from the so-called Lovelace-Shapiro (LS) amplitude [15] for the reaction $\pi + \pi \to \pi + \pi$, which had a great phenomenological success in the 60's

$$A(s,t) \sim \frac{\Gamma(1 - \alpha_\rho(s))\Gamma(1 - \alpha_\rho(t))}{\Gamma(1 - \alpha_\rho(s) - \alpha_\rho(t))}, \tag{3.14}$$

with $\alpha_\rho(s) = \frac{1}{2} + \frac{s}{a}$. This amplitude predicts equal masses for the ground vector and scalar mesons, i.e. the solution $x = 1/2$ would extend (if one were to believe the LS amplitude) to both cases. An important property of this amplitude is the presence of an 'Adler zero' [16], i.e. the fact that $A(s,t) \to 0$ as $s, t \to 0$. This is actually a consequence of Goldstone theorem so there is some logical uneasiness here since we have assumed so far that chiral symmetry is unbroken, but being the presence of the Adler zero a necessary but not sufficient condition for CSB one can find a scape route from contradiction.

The generalization of this LS amplitude to reactions of the form $\pi + A \to B + C$ was performed in [17]. The generalized LS amplitude describes the axial-vector and pseudoscalar states. It turns out that the amplitude derived in [17] predicts $x = 1$ for axial-vector and pseudoscalar mesons. This is numerically inconsistent with the sum rules (even if we include f_π) as they stand and, most importantly, with chiral symmetry restoration at high energies (see e.g. [2]). The conclusion is that LS, in spite of its undeniable phenomeno logical success, is unable to reproduce the sum rules for two point functions. This rules it out as a possible model for QCD even leaving aside the fact that it does not derive from any known string theory.

Coming back to Eqs. (3.5)-(3.8) and having ruled out the possibility of a chirally symmetric solution to the sum rules we have to look for solutions with $f_\pi \neq 0$. We know from current algebra that this implies a non-zero value for the quark condensate. However, as we have discussed previously, the numerical value for the physically relevant quantity $16\pi^2 \langle \bar{q}q \rangle^2 / a^3$ is very small, so we can still use the approximate equation $x(x - \frac{1}{2})(x - 1) \simeq 0$. If $x = \frac{1}{2}$ is ruled out because it corresponds to the chirally symmetric solution, can we perhaps consider the other two solutions $x = 0$ or $x = 1$? The first one is obviously ruled out due to sign inconsistencies. The second one is viable, but inconsistent with the sum rules unless we accept deviations from strictly linear trajectories. As we do not know how CSB distorts the linear spectrum, we will introduce these effects phenomenologically through non-linear corrections to the string-like spectrum. Obviously chiral symmetry restoration at high energies requires adopting the same value of x in all channels [2, 6] and we will consider non-linear corrections in sections 5 and 6.

4. Sum rules for linear spectrum: Scalar case

It is convenient to make the rearrangement [2]:

$$\Pi(Q^2) = 2\sum_n \frac{G^2(n)m^2(n)}{Q^2 + m^2(n)} + D_0 + D_1 Q^2$$

$$= \left[\sum_n 2G^2(n) + D_0\right] - Q^2 \left[\sum_n \frac{2G^2(n)}{Q^2 + m^2(n)} - D_1\right], \quad (4.1)$$

where the infinite constant D_0 and D_1 subtract the infinities coming from the sums over resonances. After renormalization we have

$$\Pi^{S,P}(Q^2) = \frac{Z_\pi}{Q^2}\delta_{PJ} - Q^2 \sum_{n=0}^{\infty} \frac{2G_{S,P}^2}{Q^2 + m^2 + an} = \frac{Z_\pi}{Q^2}\delta_{PJ} + \frac{2G_{S,P}^2 Q^2}{a}\,\psi\left(\frac{Q^2 + m^2}{a}\right). \quad (4.2)$$

Let us consider again the chirally symmetric case. The pion residue Z_π vanishes in this case and it will be taken into account in section 6 where the chirally broken case is considered. Performing the standard procedure (just as in the vector channels using the same notation x) one obtains the sum rules

$$\frac{N_c}{16\pi^2} = \frac{G_J^2}{a}, \quad (4.3)$$

$$\frac{\alpha_s}{8\pi}\langle G^2\rangle = -G_J^2 a \left(x^2 - x + 1/6\right), \quad (4.4)$$

$$-\frac{2\xi^J}{3}\pi\alpha_s\langle\bar{q}q\rangle^2 = \frac{2}{3}G_J^2 a^2 x \left(x - 1/2\right)\left(x - 1\right). \quad (4.5)$$

The general requirement for the spectrum is the chiral symmetry restoration at high energies (i.e. equal x for both channels). There are three solutions with these properties: $x = 0, 1/2, 1$. Substituting these solutions in Eq. (4.4) and making use of Eq. (4.3) we obtain the relation between the slope and the gluon condensate

$$\frac{\alpha_s}{\pi}\langle G^2\rangle = \frac{N_c a^2}{24\pi^2}, \qquad \text{for } x = \frac{1}{2}, \quad (4.6)$$

$$\frac{\alpha_s}{\pi}\langle G^2\rangle = -\frac{N_c a^2}{12\pi^2}, \qquad \text{for } x = 0, 1. \quad (4.7)$$

The first case is in agreement with the spectrum of the LS amplitude for the scalar channel (but not for the pseudoscalar channel) and the gluon condensate is reproduced with the correct sign this time (see discussion after Eq. (3.13)). The lightest pseudoscalar has the mass, $m_\pi^2 = a/2$, in agreement with the well known fact that if no spontaneous CSB occurs, then pion stays massive. The appearance of the other two solutions is a consequence of the fact that an analogue of Eq. (3.6) is absent in the scalar channels. The vacuum energy is evidently larger on these solutions, hence, they are unphysical. It is interesting to note that the gapless solution that appears is among the energetically unfavorable ones. The existence of this solution expresses another exact result: If a rigorously chirally symmetric theory possesses a massless pseudoscalar bound state then there is a degenerate scalar meson partner (see, e.g., [18]). The fact however that the value of $\langle G^2\rangle$ disagrees in sign with Eq. (3.13) is nevertheless another sign of the inconsistency of the chirally unbroken solution.

5. Sum rules for non-linear spectrum: Vector case

According to the discussions at the end of section 3 we are forced to introduce a non-linear spectrum. We will derive the minimal non-linear realization of the spectrum consistent with the sum rules. For that we have to make some hypotheses about what represents the leading contribution in the non-linear case. First of all we know nothing from experiment about the residues of radially excited mesons. Numerical calculations show that they differ negligibly from the linear case [2]. Thus for simplicity we do not introduce non-linear corrections for the residues of excited states and for a_1-meson as well (below we show that F_{a_1} nicely agrees with experiment without any corrections). So, except for the ρ-meson, we denote

$$F_V^2 = F_A^2 \equiv F^2. \tag{5.1}$$

We note also that the mass of ρ-meson is in the real world substantially below 1 GeV, i.e. the typical CSB scale. Consequently it is natural to expect that this meson is affected by CSB to a larger extent than other vector mesons. We shall make the hypothesis that CSB induces a shift in the residue of the ρ meson (and in no other vector or axial mesons). We shall later attempt to justify this hypothesis. Then

$$F_\rho^2 = F^2 + \tilde{F}^2. \tag{5.2}$$

Now let us introduce the non-linear mass spectrum in the form

$$m_J^2(n) = m^2 + an + \delta_J(n), \tag{5.3}$$

where the corrections always can be represented as follows:

$$\delta_J(n) = aA_J f_J(n), \qquad f_J(0) = 1. \tag{5.4}$$

Here the dimensionless constants A_J are supposed to be due to CSB, i.e. they are proportional to f_π^2/a, and $f_J(n)$ are decreasing functions of n only. They have to vanish at least exponentially in n according to [2], or polynomially according to [6] (see discussion in section 8), but in this work we will not adhere to any particular model for these functions. If the non-linear corrections provide a rather big contribution to the masses of ground states but relatively small one to those of excited states (as we expect) then the parametrization (5.4) automatically will not be very sensitive to a concrete choice of $f_J(n)$.

An important assumption of the present analysis is that the slope a has no corrections. This is a universal quantity which does not feel CSB. Any corrections to the spectrum are encoded in the value of intercept m^2 and in the non-linear contribution $\delta(n)$. Such a picture is well justified phenomenologically.

The contribution of the non-linear mass corrections to the correlator is (for the time being we denote $\delta \equiv \delta_J(n)$ and omit the general factor)

$$\frac{1}{Q^2 + m^2 + an + \delta} = \frac{1}{Q^2 + m^2 + an}\left(1 + \frac{\delta}{Q^2 + m^2 + an}\right)^{-1} =$$
$$\frac{1}{Q^2 + m^2 + an} - \frac{\delta}{Q^4} + \frac{1}{Q^6}\left(2m^2\delta + 2an\delta + \delta^2\right) + \mathcal{O}\left(\frac{1}{Q^8}\right). \tag{5.5}$$

The ρ-meson additional contribution is

$$\frac{\tilde{F}^2}{Q^2 + m^2 + aA_V} = \frac{\tilde{F}^2}{Q^2} - \frac{\tilde{F}^2 a(x + A_V)}{Q^4} + \frac{\tilde{F}^2 a^2(x + A_V)^2}{Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right). \qquad (5.6)$$

Let us introduce the constants

$$C_0^J \equiv \sum_{n=0}^{\infty} f_J(n), \qquad C_1^J \equiv \sum_{n=0}^{\infty} n f_J(n), \qquad C_2^J \equiv \sum_{n=0}^{\infty} f_J^2(n). \qquad (5.7)$$

After summation over n and making use of notations (5.1), (5.2), and (5.7) sum rules (3.5)-(3.8) become

$$\frac{N_c}{24\pi^2} = \frac{F^2}{a}, \qquad (5.8)$$

$$f_\pi^2 \delta_{AJ} + \tilde{F}^2 \delta_{VJ} = F^2 \left(x - 1/2\right), \qquad (5.9)$$

$$\frac{\alpha_s}{12\pi F^2 a} \langle G^2 \rangle = x^2 - x + 1/6 - 2\Delta_1^J, \qquad (5.10)$$

$$0 = x\left(x - 1/2\right)\left(x - 1\right) - 3\Delta_2^J, \qquad (5.11)$$

where the corrections Δ_i^J are non-zero due to CSB and they are given by

$$\Delta_1^J = A_J C_0^J + \frac{\tilde{F}^2}{F^2}(x + A_V)\delta_{VJ}, \qquad (5.12)$$

$$\Delta_2^J = A_J \left(2C_0^J x + 2C_1^J + C_2^J A_J\right) + \frac{\tilde{F}^2}{F^2}(x + A_V)^2 \delta_{VJ}. \qquad (5.13)$$

From Eq. (5.9) we get

$$\tilde{F} = f_\pi. \qquad (5.14)$$

Thus from the same equation one has

$$x = \frac{1}{2} + \frac{f_\pi^2}{F^2} = \frac{1}{2} + \frac{24\pi^2 f_\pi^2}{N_c a}. \qquad (5.15)$$

At this stage we need some additional input to advance. For reliability this input should be a quantity which is experimentally known with a good accuracy. Except f_π (which is already an input) the best candidate for such a quantity is $F_\rho = 154 \pm 8$ MeV. We note now that this number is very close to $\sqrt{3} f_\pi \approx 151$ MeV. Thus, within our accuracy, we may put $F_\rho = \sqrt{3} f_\pi$ as input. Making use of Eqs. (5.1) and (5.14) one obtains

$$F = \sqrt{2} f_\pi \approx 123 \, \text{MeV}, \qquad (5.16)$$

which is in agreement with the experimental value for the decay constant of a_1-meson (see (5.1)): $F_{a_1} = 123 \pm 25$ MeV. From Eq. (5.15) we immediately get the solution for the intercept, $x = 1$, and the relation

$$a = \frac{48}{N_c}\pi^2 f_\pi^2, \qquad (5.17)$$

in very good agreement with phenomenology. This formula happens to be the result reported in [19] based on completely different grounds. It is curious to note that relation (5.17) can be obtained from combining the slope of the LS amplitude, $a = 2m_\rho^2$ (we

remind that the slope has no corrections in the present analysis), with a known formula for the mass of ρ-meson (5.18) (both relations were also found in [5]). In principle, Eq. (5.17) could be taken as input from the very beginning, all other results are then automatically reproduced. This way would be natural in the sense that considering the CSB case we should insert a spectrum of string with CSB incorporated. Such strings were considered, e.g., in [20]. Further development of this approach resulted in a qualitative derivation of Eq. (5.17) in [19], where the scales of the Goldstone boson physics and of the string dynamics are supposedly related through Eq. (5.17) by the chiral anomaly.

It is thus quite remarkable that CSB forces a jump from $x = \frac{1}{2}$ to $x \simeq 1$. A physical interpretation of the intercept is that it is a typical mass of ground state, hence, it should be tightly related to the scale of CSB, $\Lambda_{\text{CSB}} \simeq 4\pi f_\pi$. Accidentally or not, they are identical, $m^2 \simeq a \simeq \Lambda_{\text{CSB}}^2$. A deeper understanding of this coincidence might lead to an independent derivation of Eq. (5.17).

From the last sum rule Eq. (5.11) one can see that at the solution $x = 1$ the contribution due to the linear part of the spectrum (the first term in the r.h.s.) is exactly cancelled. As a result the corrections turn out to be directly related to the (unwritten) quark condensate in the l.h.s. This is in a full agreement with our initial assumption concerning the nature of the corrections.

The slope a has a purely gluonic nature and, consequently, its value should not change dramatically after CSB. Thus, one may substitute Eq. (5.17) into Eq. (3.12) and obtain the mass of ground state in the chirally symmetric case:

$$m_J^2(0) = \frac{24}{N_c}\pi^2 f_\pi^2. \tag{5.18}$$

This is nothing but a formula for the mass of ρ-meson obtained some time ago within the Borel QCD sum rules [21] and rederived in some other types of sum rules (see, e.g., [5, 22]). This formula also usually holds when matching different models to the vector meson dominance and appears all over the literature (see, e.g., [23, 24]). Relation (5.18) reproduces the experimental mass of ρ-meson with a unexpectedly high accuracy (we remind that $f_\pi = 87$ MeV in the chiral limit). Thus, we should conclude that the mass of the ground vector meson practically does not shift after CSB. Then we immediately obtain the constant A_V parametrizing corrections in the vector channel

$$A_V = -\frac{24\pi^2 f_\pi^2}{N_c a} = -\frac{1}{2}. \tag{5.19}$$

Substituting Eqs. (5.14), (5.19), and (4.6) (which can be rewritten as $\frac{\alpha_s}{\pi}\langle G^2\rangle = F^2 a$) into Eq. (5.10) we obtain the estimate

$$C_0^V = \frac{5}{12}. \tag{5.20}$$

From the same equation for the axial-vector channel we have the relation

$$A_A C_0^A = \frac{1}{24}. \tag{5.21}$$

For a rough estimate of the a_1-meson mass we may accept a kind of universality, $C_0^A \approx C_0^V$, which leads to $m_{a_1}^2 \approx 1.1a \approx (1150\,\text{MeV})^2$. Exactly the same value was obtained in the

framework of QCD sum rules [9]. It seems that a better estimate is hardly possible within the sum rules, let alone the fact that in the present work we have accepted the large-N_c limit. This example shows that the mass corrections are indeed relatively small for the mesons with the mass above the CSB scale. Even for the a_1-meson they constitute approximately 10% which competes with the accuracy of the large-N_c counting. This is what we expected from the very beginning. Keeping in mind that the constants C_i^J are of order 1 thanks to the assumed fast convergence of functions $f_J(n)$, we could independently recover in this way the approximate solution $x = 1$ in the CSB phase from Eq. (5.11) for the axial-vector case (knowing from Eq. (5.9) that $x = 0$ and $x = 1/2$ are not solutions).

6. Sum rules for non-linear spectrum: Scalar case

As in the previous section we will be interested in the minimal non-linear realization of the spectrum consistent with the sum rules. Thus, we do not introduce non-linear corrections for the residues of excited states. Let us denote

$$G_S^2 = G_P^2 \equiv G_0^2. \tag{6.1}$$

For the ground states parametrization (2.7) should be changed

$$Z_J(0) \longrightarrow Z_J(0) + \tilde{Z}_J. \tag{6.2}$$

Here $\tilde{Z}_P$ is the π-meson residue which cannot be taken into account within parametrization (2.7) and $\tilde{Z}_S$ reflects a contribution to the residue of ground scalar state. Introducing non-linear corrections to the masses in the form (5.4) and repeating the simple calculations outlined in the previous section one arrives at the sum rules

$$\frac{N_c}{16\pi^2} = \frac{G_0^2}{a}, \tag{6.3}$$

$$\frac{\alpha_s}{8\pi}\langle G^2 \rangle = -G_0^2 a \left(x^2 - x + 1/6\right) + \tilde{Z}_J, \tag{6.4}$$

$$0 = x\left(x - 1/2\right)\left(x - 1\right) - 3(x + A_J)\left(A_J C_0^J + \frac{\tilde{Z}_S \delta_{JS}}{2aG_0^2}\right). \tag{6.5}$$

Motivated by the previous results, we assume that the linear part of the spectrum does not induce CSB. The physical solutions of the system are then $x = 1$ for the scalar channel and $x = 1$ or $x = 0$ for the pseudoscalar one. In both cases we obtain the relation

$$\tilde{Z}_P = \tilde{Z}_S = \frac{\alpha_s}{8\pi}\langle G^2 \rangle + \frac{N_c a^2}{96\pi^2}. \tag{6.6}$$

Using the current algebra result,

$$\tilde{Z}_P = 2\frac{\langle \bar{q}q \rangle^2}{f_\pi^2}, \tag{6.7}$$

and phenomenological values for the gluon condensate and the slope one can estimate the quark condensate

$$\langle \bar{q}q \rangle \approx -(170 \, \text{MeV})^3. \tag{6.8}$$

Taking into account our approximations, this estimation is selfconsistent. Combined eqs. (6.6) and (6.7) pressupose a certain relation between the vacuum condensates and hadronic parameters, which is similar to a result in [25]. For $x = 1$ one has $A_P = -1$ and the n-th pseudoscalar meson is a chiral partner for the n-th scalar meson. In the second case $A_P = 0$ and the $(n+1)$-th pseudoscalar meson is a chiral partner for the n-th scalar meson. In this case there is no non-linear correction to the pseudoscalar spectrum and the π-meson has no chiral partner.

For the scalar channel we have from Eqs. (6.5) and (6.6)

$$A_S C_0^S = -\frac{1}{8}. \tag{6.9}$$

In order to estimate the mass of the ground state we can, like in the axial-vector channel, assume an approximate universality for the non-linear contributions, $C_0^S \approx C_0^V$. Making use of Eq. (5.20) this leads to $m_S^2(0) \approx 0.7a \approx (920\,\mathrm{MeV})^2$. The relative contribution to the residue of ground state due to non-linear corrections is of order $\tilde{Z}_S/Z_S(0) = 0.1 \div 0.2$, i.e. much less than for the ρ-meson.

7. Discussions

Let us give an intuitive interpretation which lies behind the results obtained. First of all, the gluon condensate is a measure of vacuum energy ε_vac. To be more precise we remind that the vacuum energy and the nonperturbative part (n.p.) of v.e.v. $\langle G^2 \rangle$ are related by the anomaly in the trace of energy-momentum tensor [26]

$$4\varepsilon_\mathrm{vac} = \langle \Theta_\mu^\mu \rangle_\mathrm{n.p.} = \frac{\beta(\alpha_s)}{4\alpha_s} \langle G^2 \rangle_\mathrm{n.p.} + \mathcal{O}(\alpha) + \cdots. \tag{7.1}$$

The term $\mathcal{O}(\alpha)$ is the contribution of quark polarization effects. In [27] the effective potential for Θ_μ^μ was constructed in the tree approximation. It was shown that the minimum of this potential (vacuum energy) must be negative, i.e. as the β-function is negative only a positive gluon condensate provides a stable vacuum.

In our analysis it is instructive to regard mesons as the spectrum of excitations given by the effective normalized quark vacuum "potential energy" U, where

$$U \equiv -\frac{\alpha_s}{\pi} \frac{\langle G^2 \rangle}{a/(4\pi^2)}, \tag{7.2}$$

and the gluon condensate $\langle G^2 \rangle$ depends on the vacuum energy ε_vac.

Let us take the vector and axial-vector channels as an example and discuss the three solutions of the sum rules mentioned in the paper. The trivial solution corresponds to the weakly-coupled QCD where the three- and four-point gluon interactions are suppressed and we effectively have $U_\mathrm{min} = 0$ like in the QED. Since the vacuum is perturbative there are neither condensates nor resonances.

The chirally-symmetric (CS) solution corresponds to fluctuations over the perturbative vacuum. The potential energy U has a local maximum in this point, $U_\mathrm{max} = m_\rho^2$, which

results in a negative gluon condensate. As we still calculate the excitations over the zero-level, there appear the mass gap $\Delta_{\text{CS}} = m_\rho^2 = a/2$.

After the CSB the potential energy recovers its minimal value, $U_{\min} = -m_\rho^2$. This new minimum turns out to be lower than the perturbative one by the value $8\pi^2 f_\pi^2$ which is equal to m_ρ^2. As a result the mass-gap enlarges by two times, $\Delta_{\text{CSB}} = 2m_\rho^2 = a = (4\pi f_\pi)^2$. It is exactly equal to the generally accepted value for the gap squared of spontaneous chiral symmetry breaking Λ_{CSB}^2!

Let us try to understand the "special" nature of ρ-meson in the framework of presented approach. In the perturbative vacuum we always have photons which are the only massless colorless particles in the Standard Model. However, in the nonperturbative vacuum this vector particle "feels" the "deep" below $U = 0$ through its virtual quark-antiquark components. As a result the quark and antiquark in the virtual pair can acquire a dynamical (constituent) mass, converting the photon into a massive vector particle. In other words, we can observe the photon over this deep as a vector particle with the mass m_ρ. Since the height of hump over zero-level and that of deep below this level are equal (the perturbative vacuum is distorted symmetrically in the both directions), the ρ-meson mass is not shifted after the CSB. The described effect in a natural way results in the enhancement (compared to other mesons) of probability for creation of this meson from vacuum, i.e. the positive shift of ρ-meson residue. This interpretation provides an intuitive picture about the mechanism of vector meson dominance.

In the axial-vector channel the situation is completely different. The a_1-meson pole is shifted in the CSB phase supposedly due to the $\pi - a_1$ mixing, i.e. the a_1-meson acquires additional mass due to a very intensive interaction with the Goldstone bosons. The mechanism seems to be the following. In the chirally symmetric phase both masses are equal, $m_{a_1}^2 = m_\pi^2 = a/2$. In the CSB phase these particles become maximally mixed. If two particles are mixed, rotation of their fields to a physical basis is known to result in repulsion of the initial masses. If the mixing is maximal the repulsion is then also maximal and in our case leads to $m_{a_1}^2 = a/2 + a/2 = a$, $m_\pi^2 = a/2 - a/2 = 0$. This interpretation is in a full agreement with old results of current algebra [28, 29]. Namely, if the chiral symmetry were unbroken the ground states would fall into irreducible representations of $SU(2) \otimes SU(2)$ chiral group, with the masses being equal insight a given representation. After CSB these representations get mixed. In particular, the following relation holds: $m_\pi^2 \cos^2 \psi + m_{a_1}^2 \sin^2 \psi = m_\rho^2$, where ψ is a mixing angle between pseudoscalar and axial-vector representations. This angle was determined from the ρ width to be around 45°, i.e. the mixing is maximal. In the chiral limit the relation converts then into the Weinberg formula. It is interesting to note that the scalar and vector representations practically do not mix after CSB. It is another manifestation of the fact that ρ-meson mass does not shift after CSB. So it does for the ground scalar meson. In practice, however, this state could have glueball, four-quark and strange components which hamper the prediction of its mass on the base of considered sum rules.

Thus the underlying reason for the appearance of mass shifts in the CSB phase is the formation of constituent quarks which interact with the Goldstone bosons in a different way in channels with different quantum numbers. This effect is drastic for the ground

states, the higher is the radial excitation the less important is the CSB phenomenon and the chiral symmetry gets restored [6, 30].

Let us emphasize that the formulas like $m_\rho = \sqrt{8}\,\pi f_\pi$ make sense only in the CSB phase. This point sometimes causes a confusion in the literature (see e.g. [22]). The mentioned relation by no means signifies that in-medium mass of ρ-meson changes in response to changing f_π, it can not be an order parameter for phase transitions like f_π. The primordial formula is $m_\rho = \sqrt{a/2}$, i.e. this mass has a gluonic origin. What we should expect at high temperatures or baryon densities is a decreasing m_{a_1} up to the value m_ρ, the latter being unchanged in the first approximation. At the same time m_π should increase up to the same value m_ρ. Finally, all masses of ground states become equal to $\sqrt{a/2}$ in the chirally restored phase.

8. Relation to other work

Concerning the nature of non-linear corrections to the string-like spectrum we would like to mention an interesting paper by Weinberg [29]. In that approach any mass matrix can be written as the sum of chirally invariant term and chirally non-invariant one. The latter term appears due to a non-trivial matrix of axial couplings and destroys the chiral symmetry in the spectrum. Our analysis suggests a one-to-one correspondence between the chirally invariant linear part of the spectrum and the first term, and the non-linear contribution and the second term in [29]. For the higher excitations the matrix of axial couplings is expected to gradually get trivial [31] resulting in suppression of the second term. This scenario realizes an expectation (see, e.g., discussions in [32] and references therein) that restoration of chiral symmetry in the hadron spectrum is related to decoupling of Goldstone bosons from highly excited states.

In some models based on sum rules at large-N_c [4, 5, 7] the intercepts in spectrum of chiral partners are different. This discrepancy with our results (and those of [2, 6]) can be confusing. We note, however, that such type of spectra are usually obtained using some additional assumptions. In [7] both models were designed with the aim to obtain a non-zero dimension-two gluon condensate. The price to pay is, e.g., that the well-motivated relation (5.17) does not hold. In the model in [4] the expression for low-energy constant L_8 was imposed as "another one" sum rule and then fine-tuned by a special prescription for the cut-off in the infinite sums. As a result this constant comes from perturbative logarithm, i.e., it is identically zero for any finite number of resonances (lack of a smooth transition from finite number of states to infinite one). The ideas in [5] are somewhat close to those of present paper, but in that paper there is no natural transition to the chirally symmetric case and the chosen way to introduce the non-linear corrections is rather questionable.

In the literature exist two different estimations for the rate of chiral symmetry restoration. In [6] the power-law minimal fall-off was obtained from matching to OPE without any convergence requirements for the sum rules, while in [2] the fall-off was estimated to be exponential after imposing the convergence for the generalized Weinberg sum rules. At present it is difficult to say which variant is realized in nature.

Finally we would like to mention a possible relation of our work to the so-called AdS/QCD models which are believed to provide a precise correspondence between conformal gauge theories (like QCD in the ultraviolet region) and some superstring theories (see, e.g., [33] and references therein). These models can naturally incorporate chiral properties of QCD, in particular, the Gell-Mann-Oakes-Renner formula was derived in [34] and relation to some effective models with hidden gauge bosons was demonstrated in [24]. As was noticed in [6] and developed in [35] they are often dual to the Migdal approach to the QCD sum rules [36] based on Padé approximation of the two-point correlators. Thus, QCD sum rules in the large-N_c limit and AdS/QCD models seem to be complementary approaches. In this respect an interesting result was reported in [37]. Namely, at certain constraints on the infrared behavior of AdS/QCD one can obtain the linear spectrum for the vector and axial mesons with the same slope. The next successful step in this direction would be a derivation of AdS/QCD model which reproduces equal intercepts to be consistent with chiral symmetry restoration at high energies. Another challenge for the AdS/QCD approach would be derivation of the relation for the string slope, Eq. (5.17).

9. Conclusions

In the late 70's before the publication of Shifman-Vainstein-Zakharov paper [8] A. Migdal formulated the program of calculation of the whole spectrum for mesons made of light quarks as a function of some unique dimensional parameter μ: $M_i = \mu C_i$, where the factor C_i depends on the discrete input parameters of QCD such as the number of colours and flavors [36]. This formula was supposed to be possible due to small values of bare quark masses which can be neglected (except the Goldstone bosons, of course).

The approach presented in this paper is, in a sense, a concrete realization of Migdal program. Indeed, the spectrum depends on the constant f_π, the slope of trajectories, and the gluon condensate. These three parameters are related by Eqs. (4.6) and (5.17). Consequently, we may choose only one of them to parametrize the whole spectrum. The most natural choice is evidently f_π. The first correction to the spectrum comes from taking into account the quark condensate squared just as was supposed by Migdal in the same paper! The extension of this analysis to mesons with other quantum numbers and to the $SU_f(3)$ group is a rather technical work.

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